First-principles quantum transport modeling of thermoelectricity in nanowires and single-molecule nanojunctions

Branislav K. Nikolić

Department of Physics and Astronomy, University of Delaware, Newark, DE 19716, U.S.A.

http://wiki.physics.udel.edu/qttg



References

J Comput Electron (2012) 11:78-92 DOI 10.1007/s10825-012-0386-y

First-principles quantum transport modeling of thermoelectricity in single-molecule nanojunctions with graphene nanoribbon electrodes

Branislav K. Nikolić · Kamal K. Saha · Troels Markussen · Kristian S. Thygesen

RAPID COMMUNICATIONS

PHYS 824

Main page

Syllabus

* Lectures

Computing

Current events

Recent changes

Go Search

functional theory (DFT).

· What links here

· Related changes

Special pages

· Research Projects

PHYSICAL REVIEW B 84, 041412(R) (2011)

Multiterminal single-molecule-graphene-nanoribbon junctions with the thermoelectric figure of merit optimized via evanescent mode transport and gate voltage

Kamal K. Saha, Troels Markussen, Kristian S. Thygesen, and Branislav K. Nikolić^{1,*} Department of Physics and Astronomy, University of Delaware, Newark, Delaware 19716-2570, USA ²Center for Atomic-scale Materials Design (CAMD), Department of Physics, Technical University of Denmark, DK-2800 Kongens Lyngby, Denmark

http://arxiv.org/abs/1201.1665

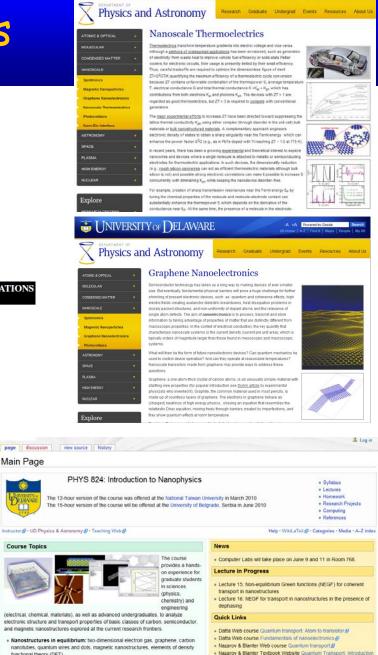
Edge currents and nanopore arrays in zigzag and chiral graphene nanoribbons as a route toward high-ZT thermoelectrics

> Po-Hao Chang and Branislav K. Nikolić Department of Physics and Astronomy, University of Delaware, Newark, DE 19716-2570, USA

> > PHYSICAL REVIEW B 81, 155450 (2010)

Electron density and transport in top-gated graphene nanoribbon devices: First-principles Green function algorithms for systems containing a large number of atoms

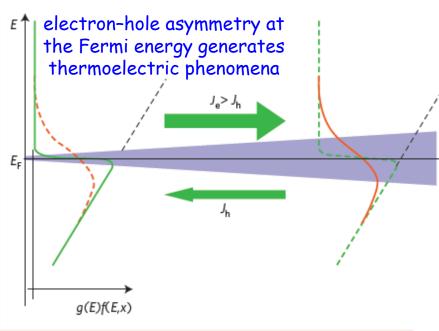
> Denis A. Areshkin and Branislav K. Nikolić Department of Physics and Astronomy, University of Delaware, Newark, Delaware 19716-2570, USA



■ UNIVERSITY OF DELAWARE

Thermoelectric Phenomena: Fundamentals and Applications

Fundamentals



$$K=K_{
m el}+K_{
m ph}$$

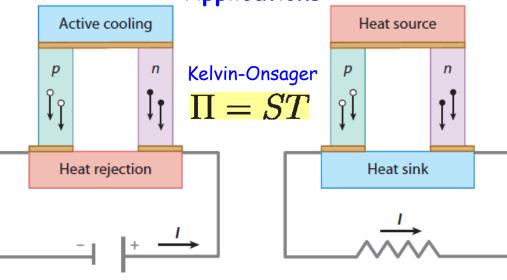
$$\kappa = \kappa_{
m el} + \kappa_{
m ph}$$



constrictions and interfaces

$$\begin{pmatrix} -\Delta V \\ \mathbf{Q} \end{pmatrix} = \begin{pmatrix} 1/G & S \\ \Pi & \kappa \end{pmatrix} \begin{pmatrix} I \\ -\Delta T \end{pmatrix}$$

Applications



Refrigeration mode

Power generation mode

Thermoelectric Figure(s) of Merit ZT in the Linear-Response Regime

$$ZT = \frac{S^2 GT}{\kappa_{\rm el} + \kappa_{\rm ph}}$$

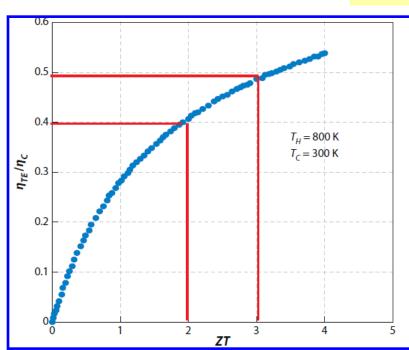
$$ZT = \frac{(S_p - S_n)^2 T}{[(\kappa_n/\sigma_n)^{1/2} + (\kappa_p/\sigma_p)^{1/2}]^2}$$

 \Box In the linear-response regime (i.e., close to equilibrium) one operates close to the small voltage V = - S Δ T which exactly cancels the current induced by the small temperature bias Δ T

 $\square \textit{As ZT} \rightarrow \infty,$ the efficiency approaches the ideal Carnot value

$$\eta_c = 1 - T/(T + \Delta T)$$

 \Box thus, in the linear-response regime $\Delta T \ll T$ typically investigated for bulk materials, the efficiency stays low $\eta_c = \Delta T / T$ even if ZT can be made very large



$$\eta_{ ext{TE}} = rac{W}{Q_H} = rac{T_H - T_C}{T_H} \left(rac{(1 + ZT_M)^{1/2} - 1}{(1 + ZT_M)^{1/2} + T_C/T_H}
ight)$$

Ultimate pragmatic goal:

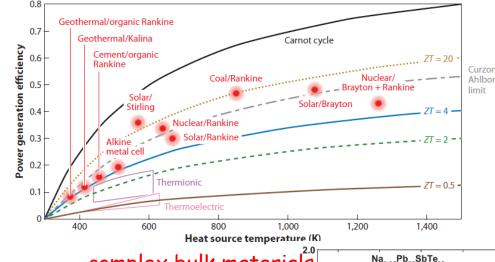
devices with $ZT \approx 2-3$ that are stable over a broad temperature range with low parasitic losses

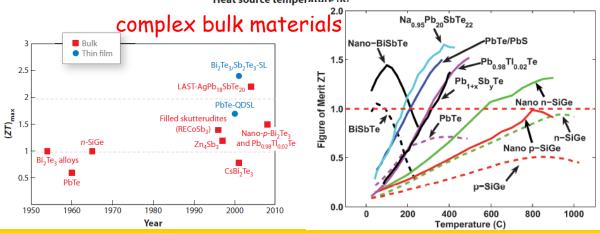
Decades of Little Progress in Increasing ZT of Bulk Materials

An inconvenient truth about thermoelectrics Nature Mater. 8, 83 (2009)

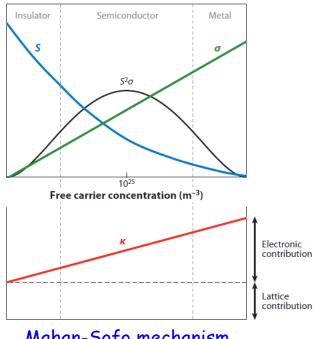


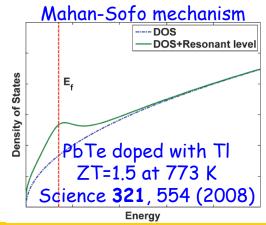
Despite recent advances, thermoelectric energy conversion will never be as efficient as steam engines. That means thermoelectrics will remain limited to applications served poorly or not at all by existing technology. Bad news for thermoelectricians, but the climate crisis requires that we face bad news head on





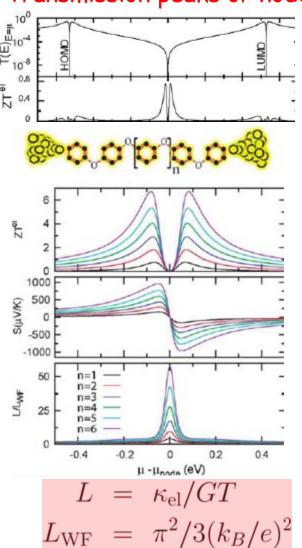
"phonon glass-electron crystal"





New Routes for ZT Optimization Brought by Low-Dimensional and Nanoscale Devices

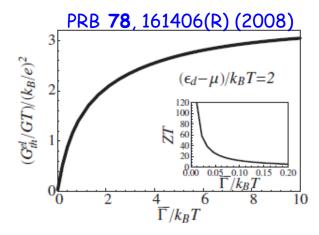
Transmission peaks or nodes



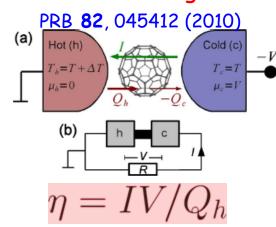
5314 (2010)

ACS Nano

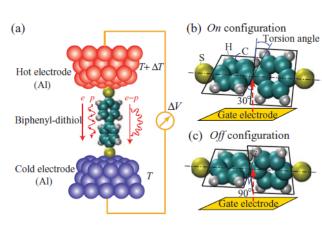
Coulomb interaction

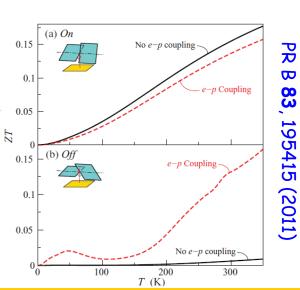


Nonlinear regime



Electron-phonon coupling



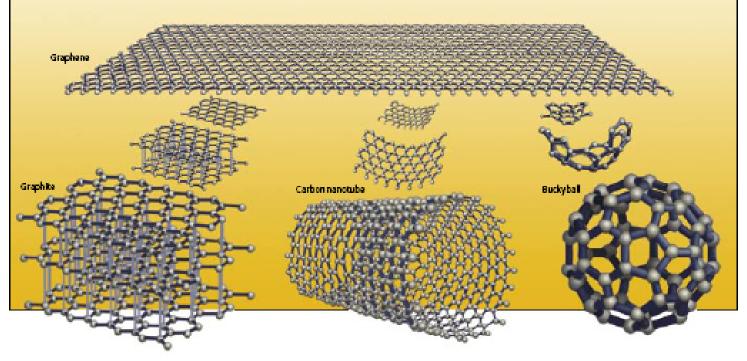


Graphene as a Building Block of Nanoscale and Low-Dimensional Devices

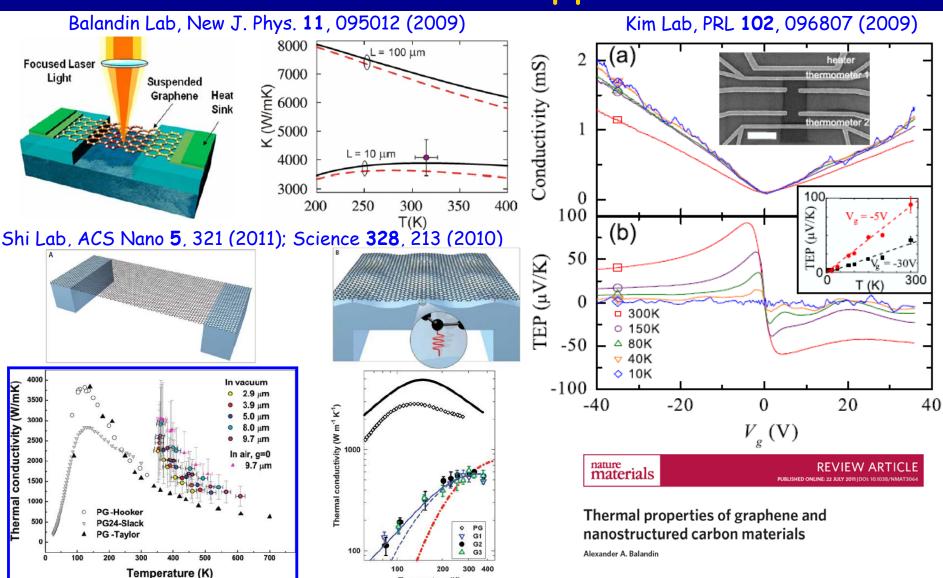
THE MOTHER OF ALL GRAPHITES

Graphene (below, top), a plane of carbon atoms that resembles chicken wire, is the basic building block of all the "graphitic" materials depicted below. Graphite (bottom row at left), the main component of pencil "lead," is a crumbly substance that resembles a layer cake of weakly bonded

graphene sheets. When graphene is wrapped into rounded forms, fullerenes result. They include honeycombed cylinders known as carbon nanotubes (bottom row at center) and soccer ball—shaped molecules called buckyballs (bottom row at right), as well as various shapes that combine the two forms.

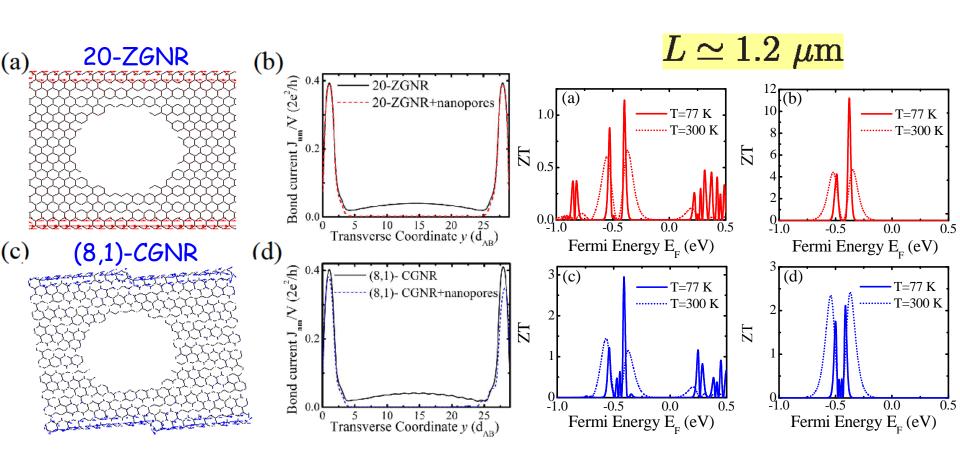


Large-Area Graphene is not Suitable for Thermoelectric Applications



Temperature (K)

Zigzag and Chiral GNRs with Nanopore Arrays as Potentially High-ZT Thermoelectrics



Nikolic group, arXiv:1201.1665

NEGF Fundamentals

☐ Basic NEGF quantities:

density of available quantum states:

$$G^r_{\sigma\sigma'}(t,t') = -rac{i}{\hbar}\Theta(t-t')\langle\{\hat{c}_{\mathbf{r}\sigma}(t),\hat{c}^{\dagger}_{\mathbf{r}'\sigma'}(t')\}
angle$$

how are those states occupied:

$$G^{<}_{\sigma\sigma'}(t,t')=rac{i}{\hbar}\langle\hat{c}^{\dagger}_{\mathbf{r}'\sigma'}(t')\hat{c}_{\mathbf{r}\sigma}(t)
angle$$

□NEGFs for steady-state transport:

$$G^r(t,t') \to G^r(t-t') \xrightarrow{\mathrm{FT}} G^r(E)$$

$$oldsymbol{D}_{ ext{eq}} = -rac{1}{\pi}\int\limits^{+\infty}\!dE\, ext{Im}\,\mathbf{G}^r(E)f(E-E_F)$$

$$G^{<}(t,t') \to G^{<}(t-t') \xrightarrow{\mathrm{FT}} G^{<}(E)$$

$$oldsymbol{D}_{
m neq} = rac{1}{2\pi i}\int\limits_{-\infty}^{+\infty} dE \, {f G}^{<}(E)$$

□NEGF (quantum) vs. Boltzmann (semiclassical) nonequilibrium statistical mechanics:

$$G^r(E) = [E - H - \Sigma_{\text{leads}}^r - \Sigma_{\text{int}}^r]^{-1}$$

 $G^{<}(E) = G^r(E)[\Sigma_{\text{leads}}^{<}(E) + \Sigma_{\text{int}}^{<}(E)]G^a(E)$

$$\mathbf{v} \cdot \nabla f + \mathbf{F} \cdot \nabla_{\mathbf{k}} f = I_{\text{coll}}[f]$$
$$\mathbf{j} = 2_s e \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \mathbf{v}(\mathbf{k}) f(\mathbf{k})$$

■NEGF-based current expression for two-terminal nanostructures:

$$I_{\alpha} = \frac{2e}{h} \int dE \, {\rm Tr} \, \left\{ \Sigma_{\alpha}^<(E) G^>(E) - \Sigma_{\alpha}^>(E) G^<(E) \right\} \quad {\rm Meir-Wingreen \,\, formula}$$

$$I(V_{ds}) = \frac{2e}{h} \int_{-\infty}^{+\infty} dE \operatorname{Tr} \left\{ \mathbf{\Gamma}_R(E, V_{ds}) \mathbf{G}_{S1}^r \mathbf{\Gamma}_L(E, V_{ds}) \mathbf{G}_{1S}^a \right\} \left[f(E - \mu_L) - f(E - \mu_R) \right]$$

Landauer-Büttiker-type formula (phase-coherent transport where Coulomb interaction is treated at the mean-field level)

Electronic Thermopower, Conductance and Thermal Conductance via NEGF

□ Electronic transmission and its integrals:

$$egin{array}{lcl} \mathcal{T}_{ ext{el}}(E) &=& \operatorname{Tr}\left\{ oldsymbol{\Gamma}_R(E) oldsymbol{G}(E) oldsymbol{\Gamma}_L(E) oldsymbol{G}^\dagger(E)
ight\} \ oldsymbol{G}(E) &=& \left[E oldsymbol{S} - oldsymbol{H} - oldsymbol{\Sigma}_L(E) - oldsymbol{\Sigma}_R(E)
ight]^{-1} \ H_{ij} &=& \left\langle \phi_i \middle| \hat{H}_{ ext{KS}} \middle| \phi_j
ight
angle, \; S_{ij} = \left\langle \phi_i \middle| \phi_j
ight
angle \ oldsymbol{\Gamma}_{L,R}(E) &=& i oldsymbol{\Sigma}_{L,R}(E) - oldsymbol{\Sigma}_{L,R}^\dagger(E) \end{array}$$

$$K_n(\mu) = rac{2}{h} \int\limits_{-\infty}^{\infty} dE \, \mathcal{T}_{
m el}(E) (E-\mu)^n \left(-rac{\partial f(E,\mu)}{\partial E}
ight)$$

□ Electronic conductance, thermopower, and thermal conductance:

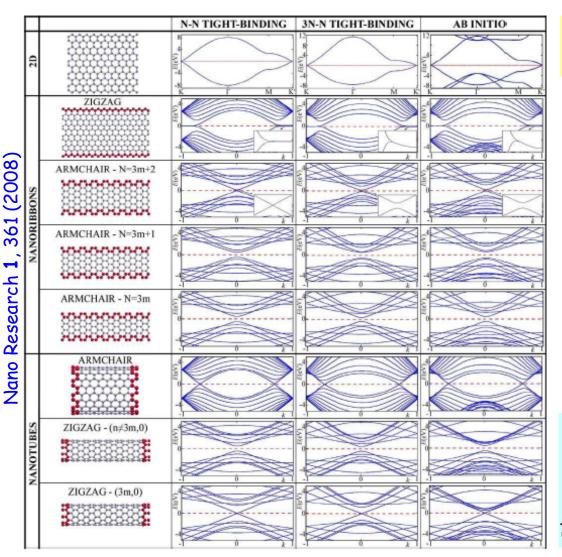
$$G=e^2K_0(\mu)$$

$$S = K_1(\mu)/[eTK_0(\mu)]$$

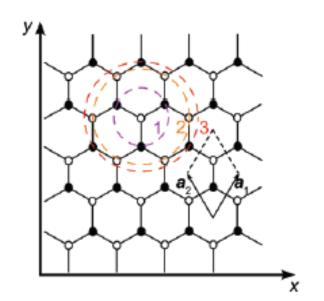
$$rac{-\partial f(E,\mu)}{\partial E}=\{2k_BT[1+\cosh(E-\mu)/k_BT]\}^{-1}$$

$$\kappa_{
m el} = \{K_2(\mu) - [K_1(\mu)]^2 / K_0(\mu)\} / T$$

Third-Nearest-Neighbor π -Orbital Tight-Binding Hamiltonian For Graphene

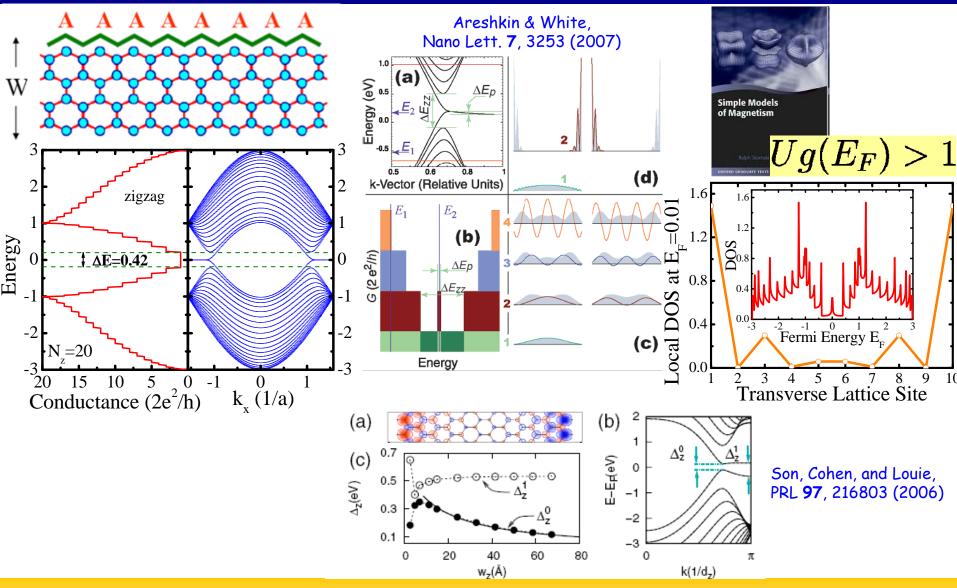


$$\hat{H} = \sum_{\mathbf{n}} arepsilon_{\mathbf{n}} \hat{c}_{\mathbf{n}}^{\dagger} \hat{c}_{\mathbf{n}} - \sum_{\mathbf{n},\mathbf{m}} t_{\mathbf{n}}^{\mathbf{m}} \hat{c}_{\mathbf{n}}^{\dagger} \hat{c}_{\mathbf{m}}$$

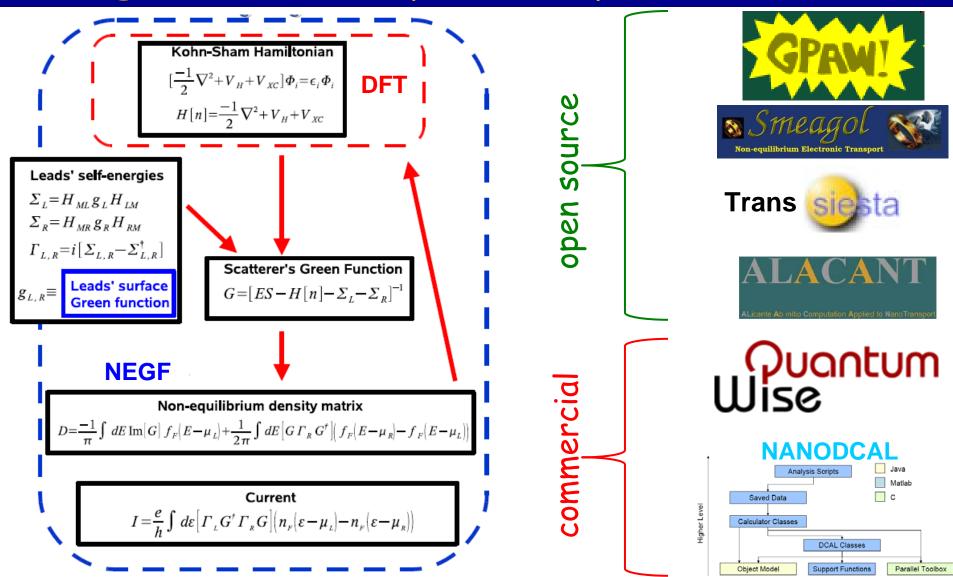


$$egin{array}{lll} t_{f n}^{{f n}+{f d}_{AB}} &=& 2.7 \ {
m eV} \ t_{f n}^{{f n}+{f d}_{AA}} &=& t_{f n}^{{f n}+{f d}_{BB}} = 0.2 \ {
m eV} \ t_{f n}^{{f n}+{f d}_{AB'}} &=& 0.18 \ {
m eV} \end{array}$$

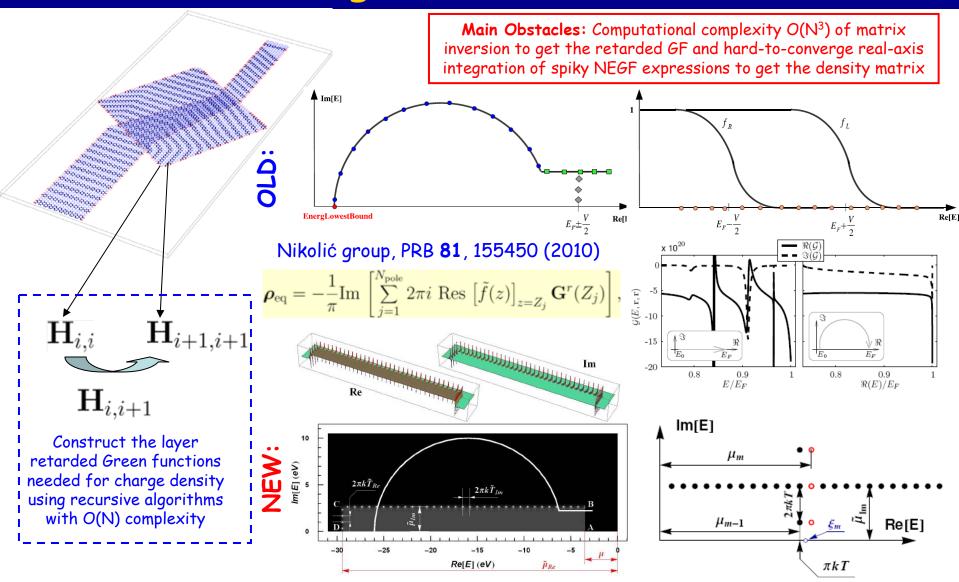
Zigzag GNR: Fundamentals



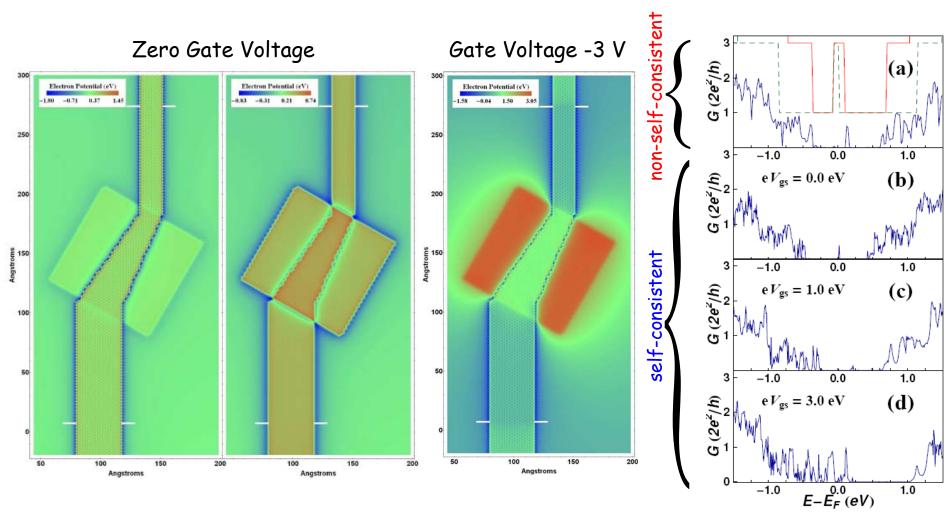
First-Principles Quantum Transport Modeling Charge, Heat and Spin Transport: NEGF+DFT



How to Apply NEGF-DFT to Devices Containing Thousands of Atoms



Gate Voltage Effect in All Carbon-Hydrogen GNRFET Composed of ~7000 Atoms



Nikolić group, PRB 81, 155450 (2010)

NEGF-DFT For Multiterminal Devices

$$D = -\frac{1}{\pi} \int_{-\infty}^{\infty} dE \operatorname{Im} \left[G(E) f(E - \mu_m) \right] + \sum_{j \neq m} \int_{-\infty}^{\infty} dE \, \rho^j(E) \left[f(E - \mu_j) - f(E - \mu_m) \right]$$

$\rho^i = G(E)\Gamma_i(E)G^{\dagger}$

$$\begin{split} \tilde{D} &= D^1 + \Delta^{12} + \Delta^{13} \\ D^1 &= -\frac{1}{\pi} \int_{-\infty}^{\infty} dE \, \mathrm{Im} \left[G(E) \, f(E - \mu_1) \right] \\ \Delta^{12} &= \int_{-\infty}^{\infty} dE \, \rho^2(E) \left[f(E - \mu_2) - f(E - \mu_1) \right] \\ \Delta^{13} &= \int_{-\infty}^{\infty} dE \, \rho^3(E) \left[f(E - \mu_3) - f(E - \mu_1) \right] \end{split}$$

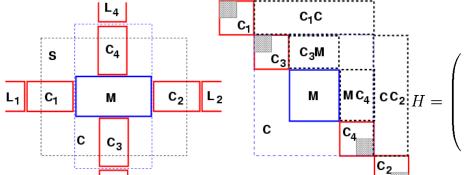
$$\tilde{D} = D^2 + \Delta^{21} + \Delta^{23} \qquad \cdots \qquad \cdots$$

$$D^2 = -\frac{1}{\pi} \int_{-\infty}^{\infty} dE \operatorname{Im} \left[G(E) f(E - \mu_2) \right]$$

$$\Delta^{21} = \int_{-\infty}^{\infty} dE \, \rho^1(E) \left[f(E - \mu_1) - f(E - \mu_2) \right]$$

$$\Delta^{23} = \int_{-\infty}^{\infty} dE \, \rho^3(E) \left[f(E - \mu_3) - f(E - \mu_2) \right]$$

$$D = w^1 \left(D^1 + \Delta^{12} + \Delta^{13}
ight) + w^2 \left(D^2 + \Delta^{21} + \Delta^{23}
ight) + w^3 \left(D^3 + \Delta^{31} + \Delta^{32}
ight)$$



$$I_{\alpha} = \frac{2e}{h} \sum_{\beta \neq \alpha} \int dE \, T(E, V_{\beta}, V_{\alpha}) [f(E - \mu_{\beta}) - f(E - \mu_{\alpha})]$$

Phonon Thermal Conductance via NEGF Coupled to Minimal Force Constant 4NNN Model

□Phonon conductance:

$$\kappa_{
m ph} = rac{\hbar^2}{2\pi k_B T^2} \int\limits_0^\infty d\omega \, \omega^2 \mathcal{T}_{
m ph}(\omega) rac{e^{\hbar\omega/k_B T}}{(e^{\hbar\omega/k_B T}-1)^2} \, rac{\mathcal{T}_{
m ph}(\omega)}{\mathbf{G}(\omega)} \, = \, \, \mathrm{Tr} \left\{ \Lambda_L(\omega) \mathbf{G}(\omega) \Lambda_R(\omega) \mathbf{G}^\dagger(\omega)
ight\} \ \mathbf{G}(\omega) \, = \, \, \left[\omega^2 \mathbf{M} - \mathbf{K} - \mathbf{\Pi}_L(\omega) - \mathbf{\Pi}_R(\omega)
ight]^{-1}$$

 \square Why no phonon-phonon scattering? $W \ll \ell pprox 677 \ \mathrm{nm}$ at 300 K [APL 98, 141919 (2011)]

□ Empirical 4NNN force constant matrix:

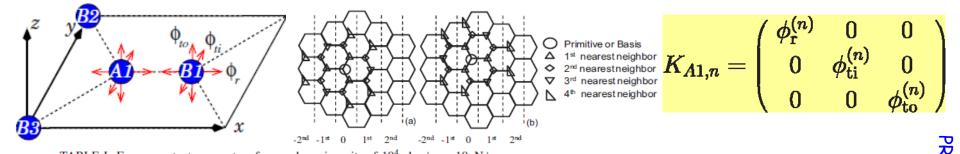
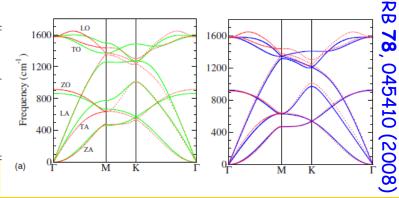


TABLE I. Force-constant parameters for graphene in units of 10⁴ dyn/cm=10 N/m.

Neighbor shell	Parameters by Saito et al. (Ref. 38)			Our parametrization		
	$\phi_{ m r}^{(n)}$	$oldsymbol{\phi}_{ ext{ti}}^{(n)}$	$\phi_{ m to}^{(n)}$	$\phi_{ m r}^{(n)}$	$\phi_{ m ti}^{(n)}$	$\phi_{to}^{(n)}$
First	36.50	24.50	9.82	41.8	15.2	10.2
Second	8.80	-3.23	-0.40	7.6	-4.35	-1.08
Third	3.00	-5.25	0.15	-0.15	3.39	1.0
Fourth	-1.92	2.29	-0.58	-0.69	-0.19	-0.55
		₁ (1)	c 1(2)	₁ (3)	111(4) (



Phonon School at IWCE 2012

First-principles thermoelectricity in nanostructures

Phonon Thermal Conductance via NEGF Coupled to Brenner Empirical Potential or DFT

□Brenner empirical interatomic potential for hydrocarbon systems (GULP or GPAW):

$$V_{ij} = f_{ij}^{C}(f_{ij}^{R} - \bar{b}_{ij}f_{ij}^{A}), \quad \bar{b}_{ij} = \frac{1}{2}(b_{ij}^{\sigma-\pi} + b_{ji}^{\sigma-\pi}) + \Pi_{ij}^{RC} + b_{ij}^{DH},$$

$$f_{ij}^{R} = \left(1 + \frac{Q}{r_{ij}}\right)Ae^{-\alpha r_{ij}}, \qquad b_{ij}^{\sigma-\pi} = \left(1 + \sum_{k \neq i,j} f_{ik}^{C}g_{ijk}\right)^{-1/2},$$

$$f_{ij}^{A} = \sum_{n}^{3} B_{n}e^{-\lambda_{n}r_{ij}}, \qquad g_{iik} = \sum_{j}^{5} \beta_{i} \cos^{i}[\theta_{ijk}].$$

$$b_{ij}^{DH} = \frac{T_{0}}{2} \sum_{k,l \neq i,j} f_{ik}^{C}f_{jl}^{C}(1 - \cos^{2}[\Theta_{ijkl}])$$

$$K_{I\alpha,J\beta} = \partial V/(\partial R_{I\alpha}\partial R_{J\beta})$$

The Brenner EIPs are short range, so they cannot accurately fit the graphene dispersion over the entire BZ. However, the thermal transport depends more sensitively on the accuracy of acoustic phonon frequencies near the zone center where the longitudinal- and transverse-acoustic (LA and TA) velocities and the quadratic curvature of the out-ofplane acoustic (ZA) branch are determined. Conversely, only weak thermal excitation of the optical phonons and acoustic phonons near the BZ boundary occurs around room temperature because of the large Debye temperature (~ 2000 K) of graphene.

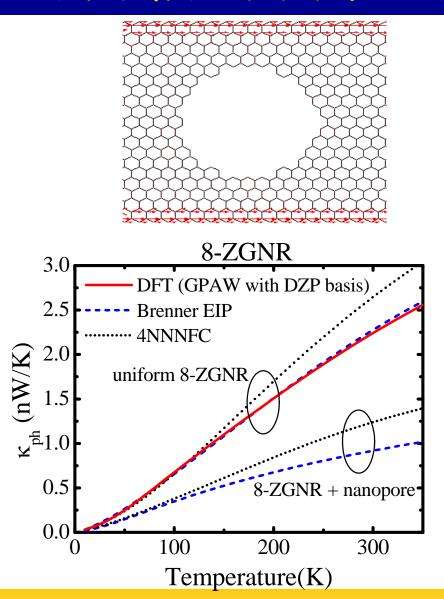
□First-principles brute force method to obtain the force constant matrix (GPAW):

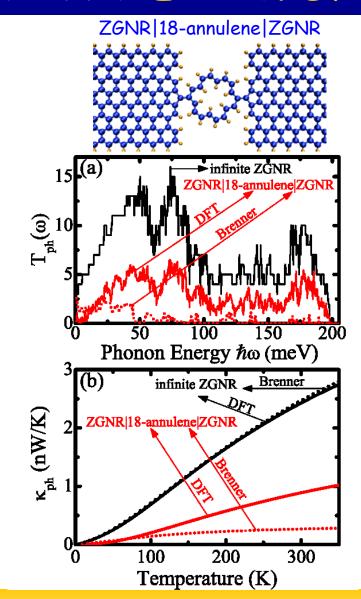
we displace each atom I by $\mathbf{Q}_{\mathbf{I}\alpha}$ in the direction α ={x,y,z} to get the forces $\mathbf{F}_{\mathbf{I}\alpha\cdot J\beta}$ on atom J≠ I in direction β

$$K_{Ilpha,Jeta}=[F_{Jeta}(Q_{Ilpha})-F_{Jeta}(-Q_{Ilpha})]/2Q_{Ilpha}$$

$$K_{Ilpha,Ieta}=-\Sigma_{J
eq I}K_{Ilpha,Jeta}$$
 for intra-atomic elements impose momentum conservation

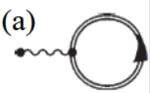
Which Method Should You Use: Minimal 4NNNFC vs. Brenner EIP vs. DFT





Coupled Electron-Phonon Transport via NEGF

$$\hat{H} = \hat{H}_{\mathrm{e}}^{0} + \hat{H}_{\mathrm{ph}}^{0} + \hat{H}_{\mathrm{e-ph}} = \sum_{i,j} H_{ij}^{0} \hat{c}_{i}^{\dagger} \hat{c}_{j} + \sum_{i,j} \hbar \omega_{\lambda} \hat{a}_{\lambda}^{\dagger} \hat{a}_{\lambda} + \sum_{\lambda,i,j} M_{ij}^{\lambda} \hat{c}_{i}^{\dagger} \hat{c}_{j} (\hat{a}_{\lambda}^{\dagger} + \hat{a}_{\lambda})$$

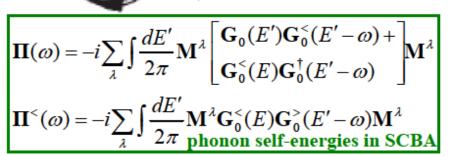


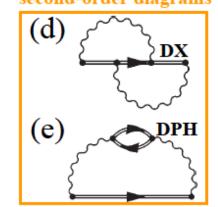
$$\Sigma^{H} = i \sum_{\lambda} \frac{2}{\omega_{\lambda}} \int \frac{dE'}{2\pi} \mathbf{M}^{\lambda} \mathrm{Tr} \left[\mathbf{G}_{0}^{<}(E') \mathbf{M}^{\lambda} \right]$$
 empirical models or **DFT (GPAW) computed**
$$\Sigma^{H,<} = 0$$

$$\mathbf{\Sigma}^{F}(E) = i \sum_{\lambda} \int \frac{dE'}{2\pi} \mathbf{M}^{\lambda} \begin{bmatrix} \mathbf{D}_{0}(E - E') \mathbf{G}_{0}^{<}(E') + \mathbf{D}_{0}(E - E') \mathbf{G}_{0}(E') \\ + \mathbf{D}_{0}^{<}(E - E') \mathbf{G}_{0}(E') \end{bmatrix} \mathbf{M}^{\lambda}$$

$$\Sigma^{F,<}(E) = i \sum_{\lambda} \int \frac{dE'}{2\pi} \mathbf{M}^{\lambda} \mathbf{D}_{0}(E - E') \mathbf{G}_{0}^{<}(E') \mathbf{M}^{\lambda} \frac{\mathbf{electron self-energies}}{\mathbf{in SCBA}}$$







□Phonon drag:

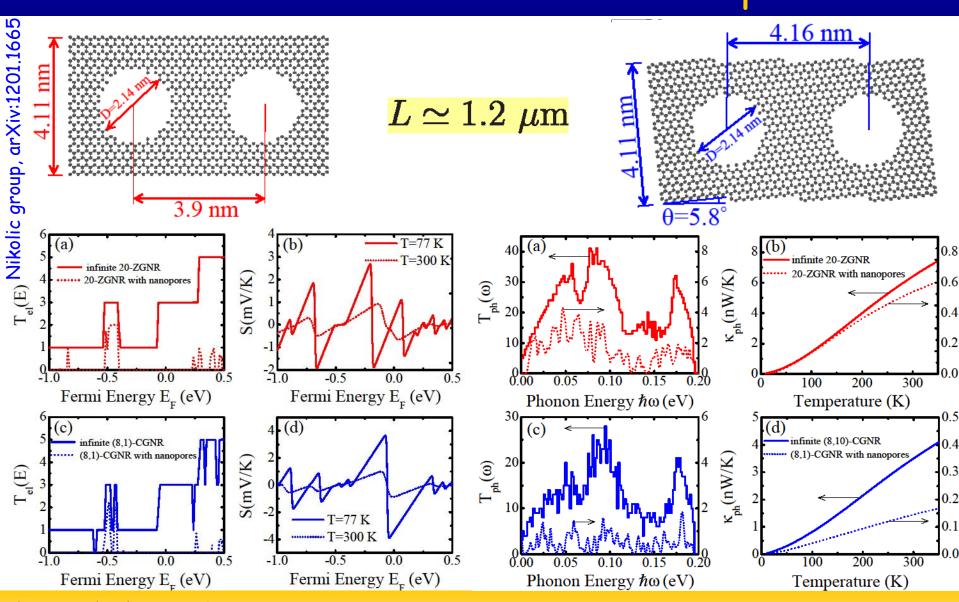
$$S = S_{
m el} + ilde{S}_{
m ph}$$

arises due to interchange of momentum between acoustic phonons and electrons

□Electron drag: phonons are dragged by electrons from low into high T region

□Three- and fourphonon many-body interactions PRB 74, 125402 (2006)

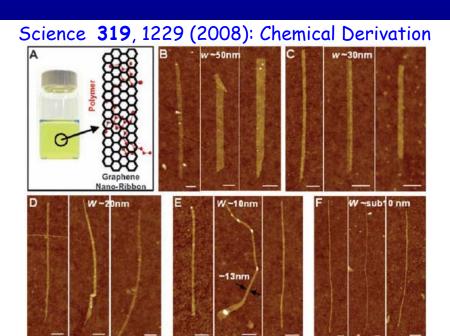
Electron and Phonon Transport in ZGNRs and CGNRs with Nanopores



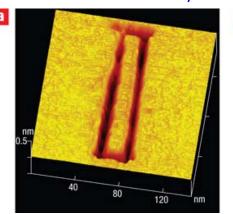
Phonon School at IWCE 2012

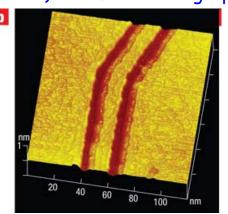
First-principles thermoelectricity in nanostructures

Graphene Nanoribbons: Fabrication



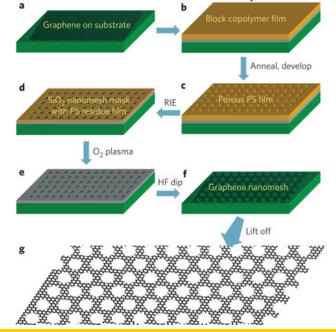
Nature Nanotech. 3, 397 (2008): STM Nanolithography







Nature Nanotech. 5, 190 (2010): Graphene nanomesh



Band vs Transport Gaps in GNRs with Rough Edges

PHYSICAL REVIEW B 81, 193408 (2010)

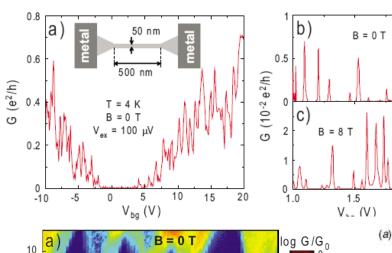
IOP PUBLISHING
Semicond. Sci. Technol. 25 (2010) 034002 (7pp)

MICONDUCTOR SCIENCE AND TECHNOLOG

doi:10.1088/0268-1242/25/3/0340

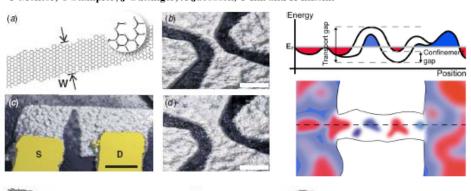
Magnetotransport through graphene nanoribbons

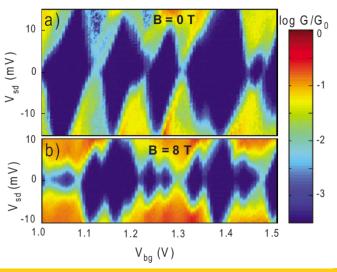
Jeroen B. Oostinga, 1,2 Benjamin Sacépé, 1 Monica F. Craciun, 3 and Alberto F. Morpurgo 1

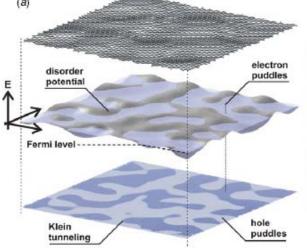


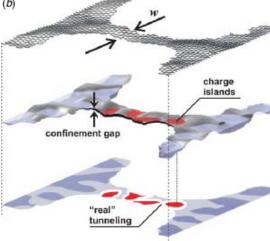
Energy and transport gaps in etched graphene nanoribbons

F Molitor, C Stampfer, J Güttinger, A Jacobsen, T Ihn and K Ensslin









Phonon School at IWCE 2012

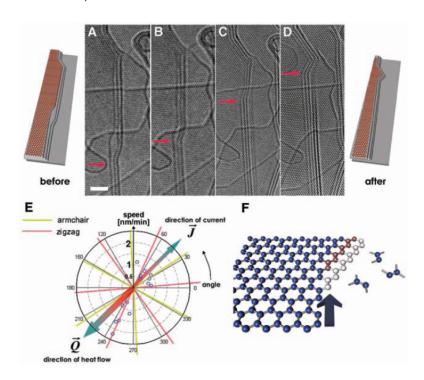
First-principles thermoelectricity in nanostructures

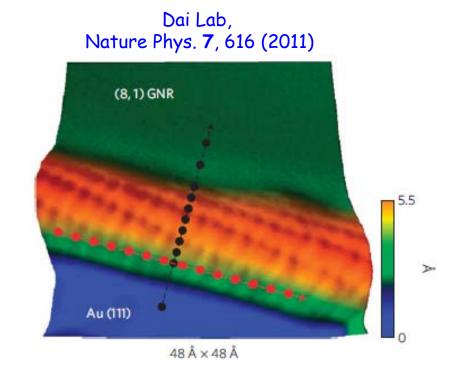
Can We Control Formation of GNR Edges?

SCIENCE VOL 323 27 MARCH 2009

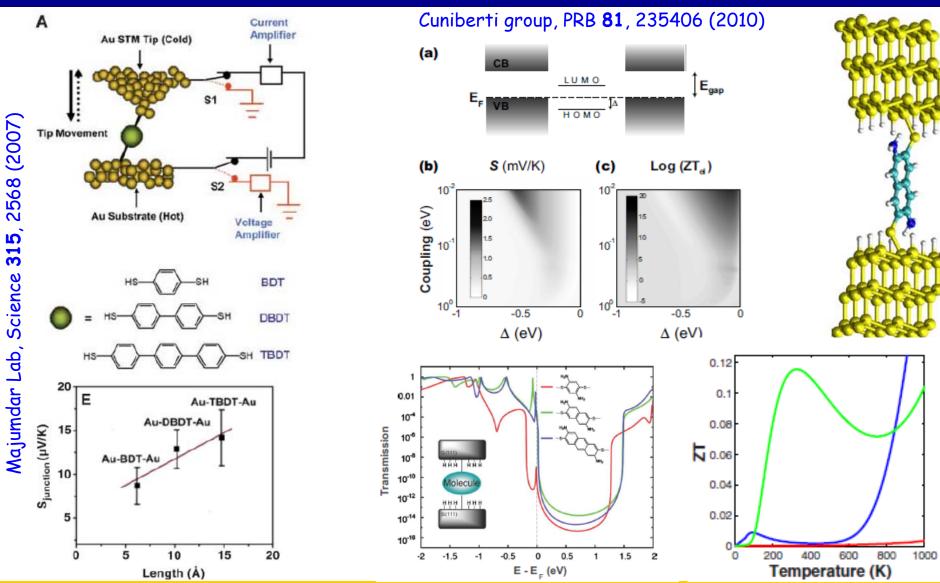
Controlled Formation of Sharp Zigzag and Armchair Edges in Graphitic Nanoribbons

Xiaoting Jia, Mario Hofmann, Vincent Meunier, Bobby G. Sumpter, Jessica Campos-Delgado, José Manuel Romo-Herrera, Hyungbin Son, Ya-Ping Hsieh, Alfonso Reina, Jing Kong, Mauricio Terrones, Mildred S. Dresselhaus, Hildred S. Dresselhaus, Hildred S. Dresselhaus, Hildred S. Dresselhaus, Mildred S.



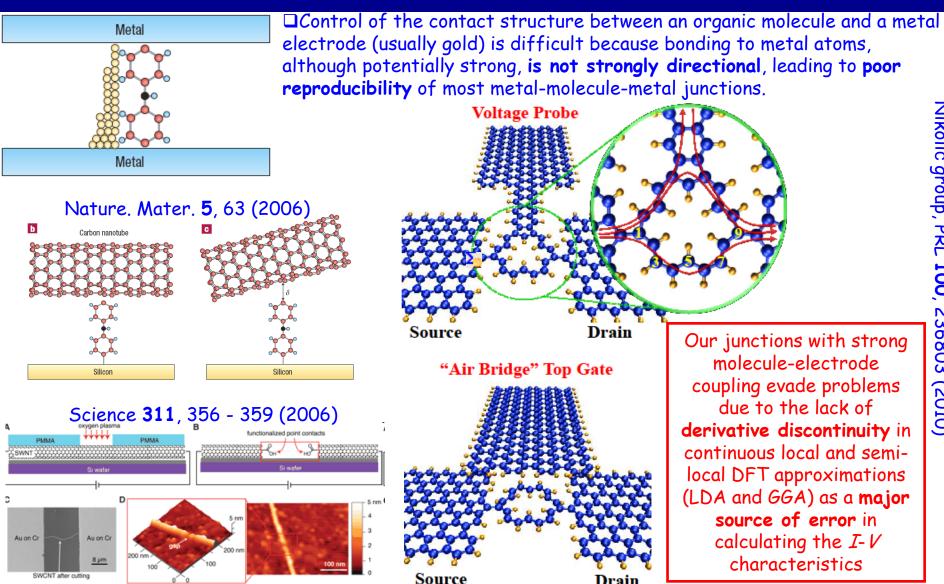


Thermoelectricity in Single-Molecule Nanojunctions (see mini-review arXiv:1111.0106)

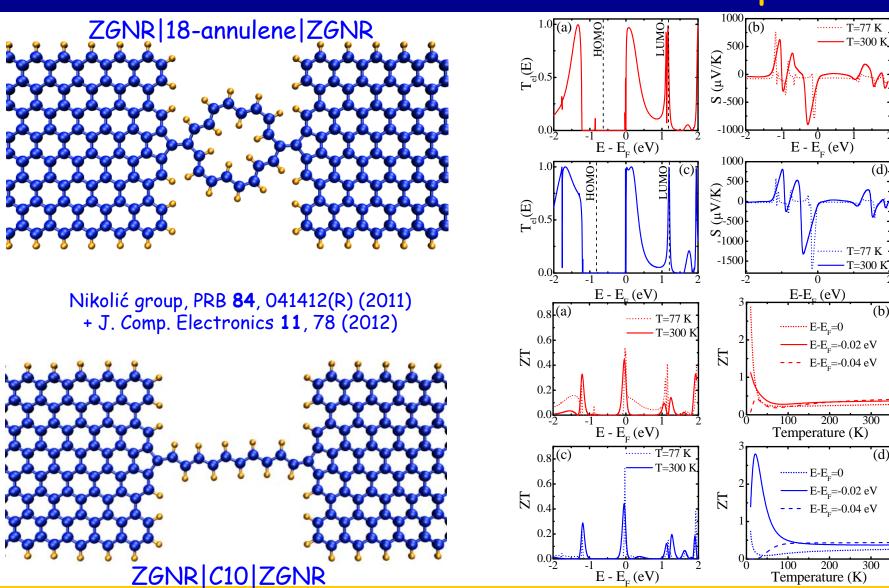


First-principles thermoelectricity in nanostructures

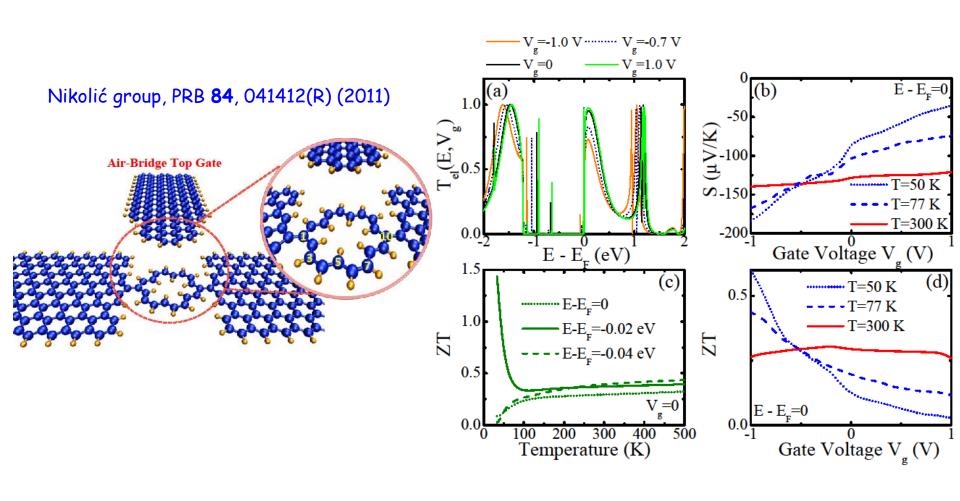
Toward Metal-Free Molecular Electronics



ZGNR molecule ZGNR Thermoelectric Devices Based on Evanescent Mode Transport

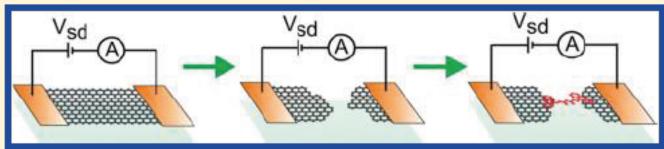


Three-Terminal Single-Molecule Nanojunction Thermoelectrics

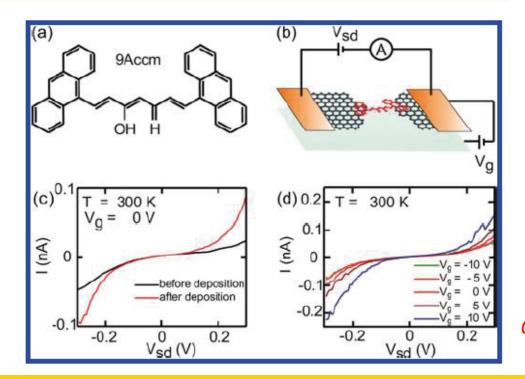


Fabrication of Single-Molecule Nanojunctions with Graphene Electrodes

van der Zant Lab, Nano Lett. 11, 4607 (2011)

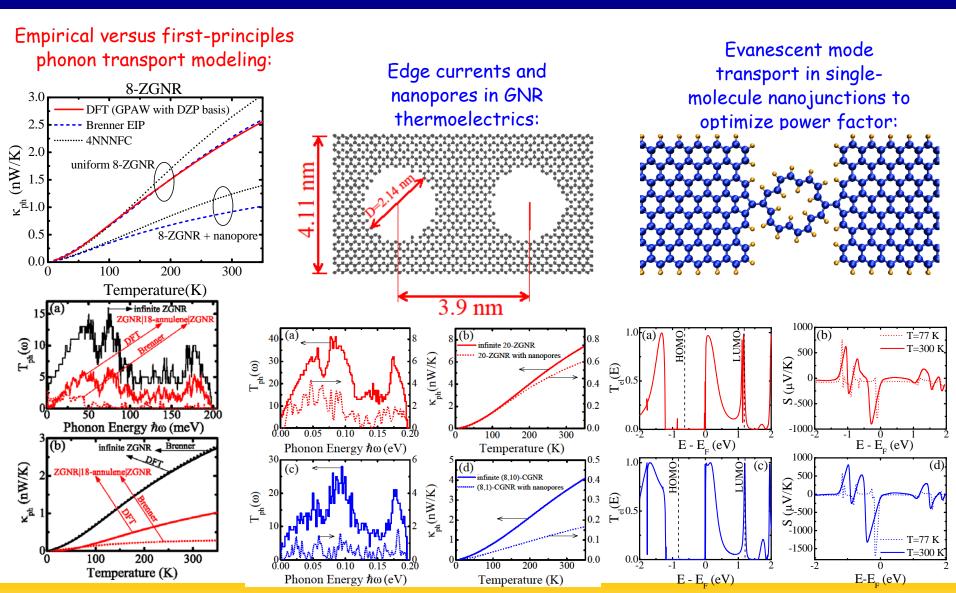


depositing molecules inside a few-layer graphene nanogap (of the size 1-2 nm) formed by feedback controlled electroburning



Gatable I-V characteristics at room temperature

Conclusions in Pictures



First-principles thermoelectricity in nanostructures